Table 1 Deployment conditions in low-pressure Mars atmospheres

Atm	Entry angle, deg	Velocity, fps	Altitude, ft
G	−90	3750	37,000
...	−40	3150	51,000
...	−20	1320	81,000
H	−90	2800	30,000
I	−90	2450	38,000
J	−90	2250	62,000
...	−60	2430	69,000
...	−40	2440	81,000

very lightly damped; hence, the sensitive axis of a body-mounted accelerometer may oscillate about the drag vector. Studies using the Apollo shape have shown that, for angle-of-attack oscillations of 20° amplitude, a damped accelerometer could read about 2% low. The desired acceleration ratio should be biased to account for the possibility of this error, depending upon the accuracy with which the angle of attack at entry can be controlled, and, therefore, the oscillations, which can be expected during the time acceleration measurements are to be made. An uncertainty in entry velocity is also to be considered when establishing the desired acceleration ratio; however, this effect is reduced with the choice of deployment criterion discussed here.

The deployment criterion given in Eq. (7) is derived for any atmospheric model with constant scale height in the region of appreciable acceleration. If landers with a high ballistic coefficient are to enter with steep entry angles, the possibility of deploying the parachute in a troposphere must be considered. In this case, the relationship between acceleration and velocity must be determined from inspection of computed trajectory data for the specific atmospheres expected. Table 1 shows the velocity and altitude at which the acceleration is 12% of peak acceleration for entry into Kaplan's low-pressure model atmospheres[2] shown in Table 2. Note that, with this criterion, deployment does not necessarily occur at the highest altitude possible for the particular atmosphere encountered; rather, a minimum altitude for a model "extreme" atmosphere is assured. It should be noted also that these represent worst-case designs; in the event that the capsule enters at a low entry angle or encounters a more dense atmosphere, this criterion will deploy the parachute at a higher altitude and provide more time for measuring and transmitting scientific data.

Table 2 Characteristics of Kaplan atmospheres

Atmosphere	*G*	*H*	*I*	*J*
Surface pressure	11	11	15	30 mb
Stratosphere temp.	234	414	324	234 °R
Surface temp.	468	468	414	378 °R
Molecular wt	42.6	42.6	38.8	31.3
Tropopause alt	82,300	19,000	33,400	64,800 ft
Inverse scale height, β	4.506	2.546	2.963	3.308 10^{-5} ft
Surface density	4.21	4.21	5.90	10.42 10^{-5} slugs/ft^3
Artificial surface density	26.40	4.89	8.44	27.55 10^{-5} slugs/ft^3

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Equilibrium Flow of an Ideal Dissociating Gas over a Cone

F. Edward Ehlers*

Boeing Scientific Research Laboratories, Seattle, Wash.

1. Introduction

FOR the ideal dissociating gas of Lighthill[1] under the assumption of thermodynamic equilibrium, the differential equations for the axially symmetric supersonic flow over a cone are derived, transformed, and simplified. Numerical results are given for flight in pure oxygen at a Mach number of 26.24 and for an ambient pressure corresponding to 100,000 ft altitude.

The equations of state of the ideal dissociating gas for the pressure p, density ρ, temperature T, enthalpy i, and fraction of dissociation α, as given by Lighthill,[1] are

$$p = \rho T(1 + \alpha) \tag{1}$$

$$i = (4 + \alpha)T + \alpha \tag{2}$$

$$\rho\alpha^2/(1 - \alpha) = e^{-1/T} \tag{3}$$

2. Equations of Motion for the Flow over a Cone

The equations of motion for the steady flow of a dissociating gas over a cone are given in Ref. 3. Eliminating the pressure by introducing the velocity of sound c, $c^2 = (dp/d\rho)_s$, the equations may be written in the form

$$d\sigma/d\omega = -v(u + v\cot\omega)/(v^2 - c^2) \tag{4}$$

$$dv/d\omega = c^2(u + v\cot\omega)/(v^2 - c^2) - u \tag{5}$$

$$du/d\omega = v \tag{6}$$

where u and v, the velocity components, and ω are described in Fig. 1; and $\sigma = \log\rho$. Differentiating the equilibrium relation, Eq. (3), logarithmically, yields

$$d\alpha = \alpha(1 - \alpha)dT/(2 - \alpha)T^2 - \alpha(1 - \alpha)d\rho/\rho(2 - \alpha) \tag{7}$$

This equation together with Eqs. (1) and (2) and the relation $di = dp/\rho$ yields

$$dT = T\varphi d\rho/\rho\psi \tag{8}$$

$$c^2 = (dp/d\rho)_s = T[2/(2 - \alpha) + \varphi^2/\psi] \tag{9}$$

where

$$\varphi = 1 + \alpha + \alpha(1 - \alpha)/(2 - \alpha)T$$
$$\psi = 3 + \alpha(1 - \alpha)/(2 - \alpha)T^2$$

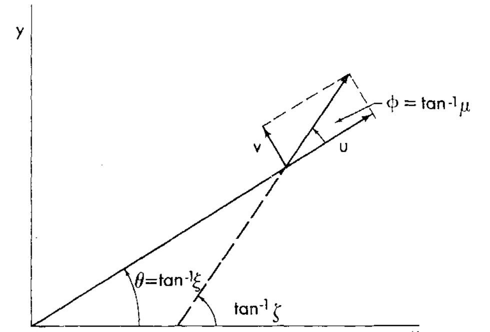

Fig. 1 Illustration of velocity vector and its components along rays through the cone vertex.

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* Staff Member, Mathematics Research Laboratory. Associate Fellow Member AIAA.

The transformation

$$u = q/(1 + \mu^2)^{1/2} \qquad v = q\mu/(1 + \mu^2)^{1/2}$$

together with

$$\beta = (q^2/c^2 - 1)^{1/2} \qquad \xi = \tan\omega$$

where q is the velocity magnitude, $\mu = \tan\Phi$, and Φ is the angle shown in Fig. 1, converts Eqs. (4–6) into a set of differential equations with rational coefficients. Substituting into Eqs. (5) and (6), solving for $d\mu/d\xi$ and $(1/q)\ dq/d\xi$, and then simplifying, yields

$$dq/d\xi = q\mu\chi \tag{10}$$

$$d\mu/d\xi = (1 + \mu^2)[\chi - 1/(1 + \xi^2)] \tag{11}$$

where $\chi = (\xi + \mu)/\xi(1 + \xi^2)(\beta^2\mu^2 - 1)$. Similarly, Eqs. (4, 8, and 7) become

$$d\sigma/d\xi = -(\beta^2 + 1)\mu\chi \tag{12}$$

$$dT/d\xi = -T\varphi(\beta^2 + 1)\psi u/\psi \tag{13}$$

$$d\alpha/d\xi = -T(1 + \alpha - 3\varphi/\psi)(\beta^2 + 1)\chi\mu \tag{14}$$

With the values of ξ, q, β, μ, σ, T, and α behind a given shock as initial values, the Eqs. (10–14) are integrated until $\mu = 0$. The resulting value of ξ is the tangent of the half-angle for the solid cone that yields the given shock. Since the angle of the flow measured with respect to the cone axis is equal to $\theta + \Phi$ from Fig. 1, the tangent of the flow angle ζ is given by

$$\zeta = (\mu + \xi)/(1 - \xi\mu)$$

3. Dissociating Nonequilibrium Flow in the Neighborhood of the Cone Vertex

Where the dissociation rate is finite, the flow may be regarded as "chemically frozen" behind the shock wave in the small region near the cone vertex. If the freestream is undissociated, then Lighthill's gas for frozen flow is equivalent to an ideal gas with the ratio of specific heats $\gamma = \frac{4}{3}$.

For the ideal gas, Eq. (11) holds, and dq/q in Eq. (10) may be expressed in terms of $d\beta/\beta$ by Bernouilli's equation, then Eq. (10) becomes

$$d\beta/d\xi = \mu\chi(\beta^2 + 1)[(\gamma - 1)\beta^2 + \gamma + 1]/2\beta \tag{15}$$

4. Flow Conditions Behind a Shock

The pressure, density, and temperature before and after the shock, denoted by subscripts, 0 and 1, may be expressed in terms of the variables ξ and ζ. Thus

$$\rho_0/\rho_1 = (\xi - \zeta_1)/\xi(1 + \xi\zeta_1) = \sigma_1 \tag{16}$$

$$p_1/p_0 = 1 + 4\sigma_0(1 - \sigma_1)/3 \tag{17}$$

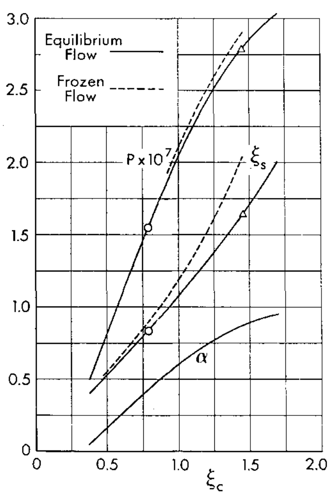

Fig. 2 The shock-wave slope ξ, dimensionless pressure p, and fraction of dissociation α for dissociating flow in equilibrium and for frozen flow against the tangent ξ_c of the solid cone half-angle for flight at a Mach number of 26.24 in an atmosphere of pure oxygen equivalent to a geometric altitude of 100,000 ft.

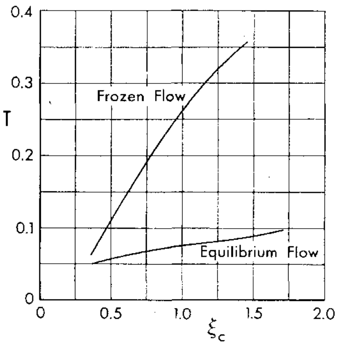

Fig. 3 Dimensionless temperature T on the cone surface plotted against the tangent ξ_c of the solid cone half-angle for flight in an atmosphere of pure oxygen at a Mach number of 26.24 and an altitude of 100,000 ft-m.

$$T_1/T_0 = \varphi_1/(1 + \alpha_1) \tag{18}$$

where $\sigma_0 = (\beta_0^2 + 1)\xi^2/(1 + \xi^2)$, and $\varphi_1 = \sigma_1[1 + 4\sigma_0(1 - \sigma_1)/3]$. Let $\varphi_2 = 1 + \sigma_0(1 - \sigma_1^2)/6$, then substituting the preceding relations into Eq. (1) and solving for α leads to

$$\alpha_1 = -[1 + T_0(\varphi_1 - 4\varphi_2)]/2 + \{[1 + T_0(\varphi_1 - 4\varphi_2)]^2 - 16T_0(\varphi_1 - \varphi_2)\}^{1/2}/2 \tag{19}$$

Rearranging the equilibrium condition, Eq. (3), in the form

$$F(\sigma_1) = \rho_0\alpha_1^2 - \sigma_1(1 - \alpha_1)\exp[-(1 + \alpha_1)T_0/\varphi_1] = 0 \tag{20}$$

yields a functional relation between σ_0 and σ_1, when α_1 is given by Eq. (19). Thus we can find σ_1 for a specified σ_0 by an iteration such as Newton's method. Since the total enthalpy i_t is constant, the quantity β is given by

$$\beta = [2(i_t - i)/c^2 - 1]^{1/2} \tag{21}$$

For frozen flow through the shock, $\alpha_1 = 0$, and Eq. (19) reduces to

$$\sigma_1 = (6 + \sigma_0)/7\sigma_0 \tag{22}$$

5. Discussion of the Calculated Results

Equations (10–14) were integrated for an atmosphere of pure oxygen at a pressure and temperature corresponding to an altitude of 100,000 ft and for a freestream Mach number of 26.24. The values of the tangent ξ of the shock angle, the dimensionless pressure p, the fraction of mass dissociation α, and temperature T, are plotted against the tangent ξ_c of the cone half-angle in Figs. 2 and 3. For comparison, the corresponding data for frozen flow are presented on the same graphs. For a given cone body, the shock angle for frozen flow is somewhat greater than that for equilibrium flow. The effect of dissociation is to reduce the temperature of the gas radically. The dot and the triangle give two data points calculated by Feldman[3] for real equilibrium air, and the agreement with the Lighthill gas is seen to be very close.

In the calculations, the density ρ and temperature T were normalized to $\rho_d = 150$ g/cm^3 and $T_d = 59000°$K as suggested for oxygen by Lighthill.[1] The pressure was normalized to satisfy Eq. (1). At 100,000-ft alt, $p_d = 2.542207 \times 10^7$ bars. The dimensionless ambient pressure and temperature are then $p_0 = 0.43831212 \times 10^{-9}$ and $T_0 = 0.00384466$. Additional data and greater detail in the derivations are given in Ref. 2, together with a formula translation listing of the computer programs.

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